Name: Feedback

SA402 · Dynamic and Stochastic Models Fall 2023 · Uhan

Quiz 8 – 11/15/2023

Instructions. You have 15 minutes to complete this quiz. You may use your plebe-issue calculator. You may not use any other materials (e.g., notes, homework, books).

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2a	0.5	
2b	0.5	
Total		/ 20

Problem 1. Two automated testing machines work together testing circuit boards. Each one is independently subject to failure. The failure rate of an automated testing machine, when it is in use, is 0.01 per hour, with the actual time to failure being exponentially distributed. The time required to repair an automated testing machine is also exponentially distributed with mean 25 hours, and only one machine can be repaired at a time. When one of the automated testing machines has failed, the other handles all of the work, which increases its failure rate to 0.02 per hour.

Model this system as a Markov process by (i) defining the state space and what the states mean, and (ii) defining the transition rates by specifying the generator matrix.

This problem is identical to Problem 3a in the Lesson 13 Exercises assigned for homework, except that the time required to repair an automated testing machine is exponentially distributed with mean 25 hours instead of 24.

Problem 2. Four Guys Burgers and Fries has 3 cashiers at its Simplexville location. Customers wait in a single queue and are served by the first available cashier, first-come first-served. The cashier area is relatively small, and can only hold 10 customers (including the 3 receiving their orders). Any customers that arrive when the cashier area is full simply go elsewhere. The average service time is 5 minutes per customer, and customers arrive at a rate of 20 per hour. Assume the interarrival times and the service times are exponentially distributed.

Model this setting as a birth-death process by answering the following prompts.

a. Define the arrival rate in each state, in terms of the number of customers per hour.

For a similar example, see Case 3 in Section 4 of Lesson 14.

Note that the system can hold at most 10 customers (including the 3 receiving their orders). So, when there are 10 customers in the system, what should the arrival rate into the system be?

b. Define the service rate in each state, in terms of the number of customers per hour.

For a similar example, see Case 4 in Section 5 of Lesson 14.

Note that when there is 1 customer in the system, only 1 of the cashiers is actively serving a customer. Similarly, when there are 2 customers in the system, only 2 of the cashiers are actively serving customers.